Homework 4

AMS 20: Mathematical Methods for Engineers

Due Tuesday August 27, 2019

Name: _____ Student ID: _____

Homework assignments will count for 25% of your overall grade. Attach extra paper as needed. Show all of your work for full credit.

1. **[20pts] Matrix Exponential.** Consider the following linear system.

$$\vec{x}\,'(t) = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \vec{x}(t)$$

   (a) **[10 pts]** Find the general solution to the ODE.

   (b) **[10 pts]** Use part (a) to determine the matrix exponential e^{At}.

2. **[25pts] Matrix Exponential.** Consider the following linear system.

$$\vec{x}\,'(t) = \begin{bmatrix} 3 & -2 \\ 0 & 3 \end{bmatrix} \vec{x}(t), \qquad \vec{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

   (a) **[5 pts]** Show that matrix A satisfies $(A - rI)^k = 0$ for some number r and some positive integer k.

   (b) **[10 pts]** Use part (a) to determine the matrix e^{At}. Hint: Use the formula

$$e^{At} = e^{rt}e^{(A-rI)t} = e^{rt}\left(I + (A-rI)t + (A-rI)^2\frac{t^2}{2!} + \cdots + (A-rI)^{k-1}\frac{t^{k-1}}{(k-1)!}\right)$$

   (c) **[10pts]** Compute the unique solution to the ODE using: $\vec{x}(t) = e^{A(t-t_0)}\vec{x}_0$.

3. **[15 pts] Non-Homogeneous Matrix-Vector ODEs.** Find the general solution to the following linear system.

$$\vec{x}\,'(t) = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} t^{-1} \\ 2t^{-1} + 4 \end{bmatrix}$$

4. **[20pts]Non-Homogeneous Matrix-Vector ODEs.** Find the unique solution to the following homogeneous matrix-vector IVP.

$$\vec{x}\,'(t) = \begin{bmatrix} 5 & -4 & 2 \\ -2 & -2 & 2 \\ 4 & 2 & 2 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 2t \\ e^{3t} \\ -5 \end{bmatrix}, \qquad \vec{x}(0) = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

5. **[20pts] Non-Homogeneous Matrix-Vector ODEs.** Find the unique solution to the following homogeneous matrix-vector IVP.

$$\vec{x}\,'(t) = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 6t^2 \\ \sec(2t) \end{bmatrix}, \qquad \vec{x}(0) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$