

HOMWORK 3

AMS 20: Mathematical Methods for Engineers

Due Tuesday August 20, 2019

Name: _____

Student ID: _____

Homework assignments will count for 25% of your overall grade. Attach extra paper as needed. Show all of your work for full credit.

1. [10pts] **Rewriting ODEs in Matrix-Vector Format.** Write the ODE as a linear system in the matrix-vector form: $\vec{x}'(t) = A\vec{x}(t) + \vec{g}(t)$.

$$\frac{d^4 y}{dt^4} + 11 \frac{d^3 y}{dt^3} + \frac{1}{2} \frac{dy}{dt} + 6y = t^2 + \ln(t)$$

2. [30pts] **Homogeneous Matrix-Vector ODEs.** Consider the following second order ODE IVP.

$$2u'' - u' - 6u = 0, \quad u(0) = 1, \quad u'(0) = -3$$

- (a) [10 pts] Write the IVP as a linear system in the matrix-vector form: $\vec{x}'(t) = A\vec{x} + \vec{g}(t)$ with the initial condition $\vec{x}(0) = \vec{x}_0$.
- (b) [20 pts] Compute the unique solution to the linear system.

3. [20 pts] **Homogeneous Matrix-Vector ODEs.** Find the general solution to the following linear system.

$$\vec{x}'(t) = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix} \vec{x}(t)$$

4. [20pts] **Homogeneous Matrix-Vector ODEs.** Find the unique solution to the following homogeneous matrix-vector IVP.

$$\vec{x}'(t) = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix} \vec{x}(t), \quad \vec{x}(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

5. [20pts] **Homogeneous Matrix-Vector ODEs.** Find the unique solution to the following homogeneous matrix-vector IVP.

$$\vec{x}'(t) = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix} \vec{x}(t), \quad \vec{x}(\pi) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$