Homework 1

AMS 20: Mathematical Methods for Engineers

Due Tuesday August 6, 2019

Name: _____ Student ID: _____

Homework assignments will count for 25% of your overall grade. Attach extra paper as needed. Show all of your work for full credit.

1. **[20pts] Separable ODEs.** Consider the following ODE.

$$\frac{dy}{dx} = 2\cos(x)\sqrt{y+1}$$

   (a) (5pts) Determine the order of the ODE and state whether it is linear or nonlinear.

   (b) (15pts) Find the general solution to the ODE.

2. **[30pts] Integrating Factor.** Consider the following IVP.

$$\frac{1}{t}\frac{dy}{dt} - \frac{2y}{t^2} = te^{-7t} \qquad y(1) = 0$$

   (a) (10pts) Determine the interval on which the solution to the IVP is valid.

   (b) (20pts) Find the solution to the IVP using an integrating factor.

3. **[10pts] Integrating Factor.** Find the general solution to the ODE using an integrating factor.

$$\frac{du}{dx} = \frac{u - x^2}{4}$$

4. **[20pts] Exact ODEs.** Determine whether the following ODE is exact. If the equation is exact, find the solution to the IVP.

$$2xy - \sec^2(x) + (x^2 + 2y)y' = 0 \qquad y(0) = 2$$

5. **[20pts] Application.** The UCSC police are called to a crime scene in a dorm room where a murder victim has been found. When they arrive at 10:00AM, the temperature of the body is taken to be 82°F. One hour later, the temperature of the body is measured again and found to be 75°F. Use Newton's law of cooling to find the time of death if the ambient temperature of the dorm room is 70°F.

   Newton's law of cooling states that the temperature T of an object changes according to the following differential equation

$$\frac{dT}{dt} = -\kappa\big(T(t) - \alpha\big),$$

   where α is the ambient temperature, κ is the cooling constant, and t is time measured in hours. (Hint: First, find T(t), then find t when the temperature of the body is a normal 98.6°F.)